Calculation and optimization of a birefringent filter for a cw dye laser

B. V. Bonarev and S. M. Kobtsev

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Rigorous design formulas are derived for the transmission coefficient of an arbitrary birefringent filter (BF) in an optical resonator. Variations of the free spectral range and the BF tuning factor with the angle between the optic axis and the plane of the phase plate are given. It is shown that for a three-element BF, the ratios of plate thicknesses of 1:3:11 and 1:4:13 are optimum. A general formula is found for the angle of tilt at which the BF transmission function has the highest contrast. Conditions are determined for the selection of BF parameters. A method for broadening the spectral tuning band of the BF is proposed. Use of these results in BF design allows us to improve its characteristics significantly.

From the time of the first report on the use of a birefringent filter (BF) for controlling the lasing spectrum of a cw dye laser,[1] several papers have appeared devoted to the analysis of the selective properties of such filters.[2–5] This type of wavelength selectors are widely used in tunable lasers (see the survey of the application of BF in Ref. 7) owing to the establishment of the fundamental characteristic of the BF [the contrast transmission function, a constant (accurate up to 10%) filter tuning coefficient in a spectral width of 100 nm (Ref. 6)], and also the absence of dielectric coatings and simplicity of control.

Usually BF represent one or a few phase plates, situated in the resonator at the Brewster angle with respect to the incident beam. The optic axes of all the plates are parallel to one another and make a certain angle with the plane of incidence. The wavelength of the radiation is tuned by matched rotation of the plates without changing their tilt. The angle β between the optic axis OA (Fig. 1) and the plane of the plate is chosen to be equal to 0 or 25°, and the ratios of the plate thicknesses in the most common three-element filters consist of a multiplicity of variations—1:2:9,[3] 1:2:10,[8] 1:2:15,[3,5] 1:4:16[2,6] and others. Filters having identically thick thinnest plates (we will call them first-order plates), but different angles β and plate thickness ratios, differ in the free spectral range, the width of the fundamental and the height of the secondary transmission peaks, and the angular dispersion. Question of the dependence of these important BF characteristics both on its parameters, the angle β and the plate thickness ratio is clarified in the literature, but far from being complete.

To clarify this question, we must know the most general analytical or computational formula for the filter transmission coefficient, which, as has been noted in Ref. 7, is unknown at this time. Thus, the question of an optimum BF for various applications remains unsolved.

Ther derivation and analysis of computational formula for the transmission coefficient of an arbitrary birefringent selector is the purpose of this paper. Particular attention is given to the determination of the optimum orientation of the filter from the point of view of its selecting power. Criteria are proposed for the choice of the BF parameters and its design.

BIREFRINGENT FILTER TRANSMISSION FUNCTION

The Jones method[9] is used for BF description. Each (jth) phase plate situated at the Brewster angle is represented by a matrix

$$M_j = \begin{pmatrix} t^2(\cos^2\alpha + e^{-i\delta_j}\sin^2\alpha) & t\cos\alpha\sin\alpha(1 - e^{-i\delta_j}) \\ t\cos\alpha\sin\alpha(1 - e^{-i\delta_j}) & \sin^2\alpha + e^{-i\delta_j}\cos^2\alpha \end{pmatrix},$$

where t^2 is the transmittance of the Brewster surface for the energy flux of the s-polarized wave, α is the angle between the plane of incidence and the major plane of the birefringent plate, δ is the phase difference between the extraordinary and ordinary waves. Brewster surfaces of the dye solution stream are taken into account by a matrix of the form

$$M_s = \begin{pmatrix} t'^2 & 0 \\ 0 & 1 \end{pmatrix},$$

where $t' \simeq t$. Birefringent filter transmission is determined by the largest eigenvalue of the matrix M, obtained by multiplying the Jones matrices M_j^{-1} and M_s^{-1} in the sequence in which light traverses that corresponding to optical elements in a pass or bypass of the resonator. The eigenvalues $p_{1,2}$ of the matrix $M \equiv [m_{ik}]$ are found as the roots of the character-

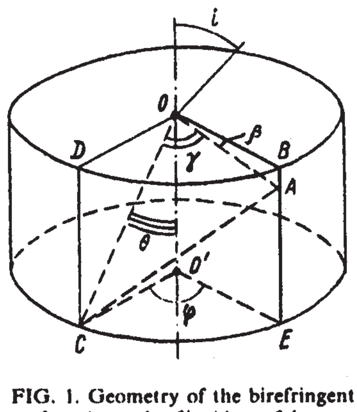

FIG. 1. Geometry of the birefringent plate. OO'—normal to the plate surface; i—angle of incident of the ray; $ODOC'$—plane of incidence of the ray; θ—angle of refraction of the extraordinary wave; γ—angle between normal of the extraordinary wave and the optic axis OA; OB—projection of optic axis on the plate surface; OAC—principal plane of the birefringent plate.

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istic equation

$$\det(M - pE) = p^2 - p(m_{11} + m_{22}) + m_{11}m_{22} - m_{12}m_{21} = 0. \qquad (1)$$

Since the matrix elements m_{ik} are complex, we introduce the notation $m_{11} + m_{22} = a + ib$, $m_{11}m_{22} - m_{12}m_{21} = c + id$. Let $p = p' + ip''$. The solution sought of the set of equations obtained after dividing Eq. (1) into real and imaginary parts, has the form

$$p = \frac{1}{2}\left[Q + a + i\left(b\left(1 + \frac{a}{Q}\right) - \frac{2d}{Q}\right)\right], \qquad (2)$$

$$Q = \sqrt{2\left[\left(R^2 + \left(\frac{1}{2}ab - d\right)^2\right)^{1/2} - R\right]}, \quad R = \frac{1}{4}(b^2 - a^2) + c.$$

The transmittance of a three-element BF for light intensity, for instance in a pass of the linear resonator of the dye laser is equal to $T = p'^2 + p''^2$, where p' and p'' are derived from Eq. (2) for the matrix $M = M_sM_1M_2M_3M_3M_2M_1M_s$. We will bear in mind this coefficient when we speak of BF transmission.

The phase delay δ_j is determined by the expression[10]

$$\delta_j = \frac{2\pi h_j}{\lambda}\left(\sqrt{n^2 - \sin^2 i} - \sqrt{n_0^2 - \sin^2 i}\right),$$

where h_j is plate thickness, i is the angle of incidence of the beam, n and n_0 are indices of refraction of the extraordinary and ordinary waves, λ is the wavelength of the radiation. For $\beta = 0$, n is found from the formula

$$n = \left[n_e^2 + \left(1 - \frac{n_e^2}{n_0^2}\right)\cos^2\varphi \sin^2 i\right]^{1/2}, \qquad (3)$$

where φ is, in general, the angle between the plane of incidence and projection of the optic axis onto the surface of the phase plate. The major indices of refraction $n_e(\lambda)$ and $n_0(\lambda)$ in Eq. (3) are computed using the cubic s-plane functions, interpolating known values of $n_e(\lambda_k)$, $n_0(\lambda_k)$ for crystalline quartz[11] in the 410–845-nm wavelength range. For $\beta \neq 0$, n is found by the secant method[12] from the equation

$$f(n) \equiv n^2 - \frac{n_e^2 n_0^2}{n_0^2 + (n_e^2 - n_0^2)\cdot\cos^2\gamma(n)} = 0, \qquad (3a)$$

where γ is the angle between the optic axis of the crystal and the normal of the extraordinary wave. We can find the function $\gamma(n)$ in Ref. 6

$$\cos\gamma(n) = \frac{1}{n}\cos\varphi\cos\beta\sin i + \left(1 - \frac{\sin^2 i}{n^2}\right)^{1/2}\sin\beta.$$

Rapid convergence of the iteration process is achieved using n_e as the initial approximation.

These formulas provide the possibility of performing rigorous numerical calculations of the BF transmission function, implemented in the Electronics-60 computer program in PASCAL. The application of complex matrix multiplication in the program simplifies the calculations considerably, and allows us to modify the model of the optical scheme easily. We should mention that we can derive from Eq. (2) an analytic expression for the transmission function of a single-element BF, such as a phase plate in a ring dye laser.

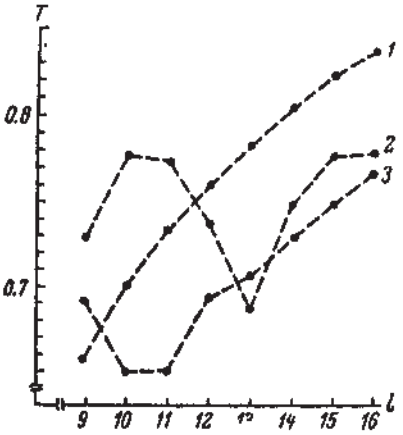

FIG. 2. Transmittance of the highest secondary peak of a three-element BF vs the plate thickness ratio. 1—1:2:l; 2—1:4:l; 3—1:3:l.

PLATE THICKNESS RATIO OF A THREE-ELEMENT BF

The secondary transmission peaks of a multielement BF can compete with the main peak, at which lasing takes place by tuning to the wing of the gain line. In a number of cases, this does not allow us to continuously tune the wavelength of the laser radiation in the entire gain band of the dye due to lasing at the secondary peak. We can reduce the height of the secondary transmission peaks, by inserting in the laser resonator additional Brewster surfaces or by the selection plate thickness ratio. First attempts in this direction are made in Ref. 3, where transmission of the highest secondary peak for three BF with different plate thickness ratio is computed. However, it is impossible to take the 1:2:9 filter found with minimum transmission to be the optimum, since it is significantly inferior to the other two filters in the width of the main transmission peak (for identical first plates).

To optimize plate thickness ratio, taking into account all the characteristics dependent on it, we compute transmission of the highest secondary peak of the 1:r:λ filters, where $r = 2$–4, and $l = 9$–16. Computed results are given in Fig. 2. The 1:3:10 and 1:3:11 filters possess minimum secondary transmission peaks. Transmission on the highest secondary peak for the 1:3:10 and 1:3:11 filters is 65%, 67% for the 1:2:9 filter, and 69% for the 1:4:13 filter. The data obtained allow us to conclude that the 1:3:11 and 1:4:13 filters are the most optimum of all the filters studied.

We note that it is recommended in Ref. 3 to use filters with thickness ratio of 1:2 for the two first plates. Figure 2 shows that it is inadvisable to use BF with a 1:2:l ratio for $l > 10$.

OPTIMUM ORIENTATION OF THE BIREFRINGENT FILTER

BF possesses the highest contrast transmission function $C = 1/T_b$, where T_b is the background transmission, when the plane of incidence is at an angle of $\alpha = 45°$ to the major planes of the phase plates.[4,10] The angle φ is related to α by the expression $\tan\varphi = \tan\alpha \sin i$, as given in Ref. 4 for the case of $\beta = 0°$. Here i is the Brewster angle. It is of interest to establish the dependence of α on φ for $\beta \neq 0°$. We can find the

TABLE I.

β (deg)	φ_{opt} (deg)
0	40.0
5	38.0
10	35.8
15	33.6
20	31.3
25	28.8
30	26.1

angle α from the cosine theorem for the trihedral angle with edges OA, OD, and OC (Fig. 1)

$$\cos \widehat{AOD} = \cos\gamma \cos(90^\circ - \theta) + \sin\gamma \sin(90 - \theta) \cdot \cos\alpha. \quad (4)$$

We have the relation for the trihedral angle with edges OB, OA, and OD

$$\cos \widehat{AOD} = \cos\varphi \cos\beta + \sin\varphi \sin\beta \cos 90^\circ. \quad (5)$$

We find from Eq. (4) and (5)

$$\cos\alpha = \frac{\cos\varphi \cos\beta \cos\theta - \sin\theta \sin\beta}{\sqrt{1 - (\cos\varphi \sin\theta \cos\beta + \cos\theta \sin\beta)^2}}. \quad (6)$$

It follows from Eq. (6) that for a fixed angle φ, for example, 40°, change of β from 0 to 50° leads to variation of α from 45° to 90°. Thus, the angle β exerts substantial effect on the interdependence of the angle φ and α. In the case of $\alpha = 45°$, Eqs. (4) transforms into

$$\cos\varphi_{opt} = \frac{\sin\beta \sin i \cos i + \sqrt{2\cos^2\beta - \cos^2 i}}{(2 - \cos^2 i)\cos\beta}. \quad (7)$$

The law of refraction and Brewster's formula in the approximation on $n \simeq \tan i$ are used in the derivation here. Results of calculating the optimum angle φ_{opt} from Eq. (7) are given in Table I (the angle i was taken to be 57°).

It is clear that the BF with $\beta = 25°$ possesses the best selectivity at $\varphi = 28.8°$. Consequently, the BF with $\beta = 25°$ should have a transmission peak at the center of the tuning band for $\varphi = 28.8°$, and not at $\varphi = 45°$, as was computed in Refs. 2, 3, 6. Deterioration of the contrast transmission function of the BF for departure of the angle φ from the optimum position is shown in Fig. 3. The graphs demonstrate sensitivity of the shape of the BF transmission spectrum to the tilt of the filter. We should note that the height of the secondary peaks is also minimum at $\varphi = \varphi_{opt}$, and not at $\varphi = 45°$, as is shown in Ref. 3. If the gain is 73% by turning the filter 10° from the optimum position, then the increase in height of the highest secondary peak is only 5%.

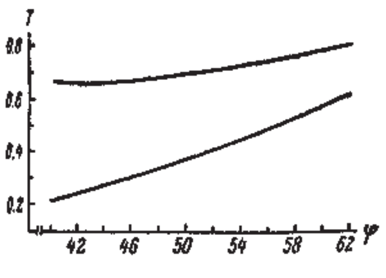

FIG. 3. Transmittance of the highest secondary peak (upper curve) and background transmittance T_b vs the angle φ for a BF with $\beta = 0°$ and plate thickness ratio 1:2:9.

EFFECT OF THE ANGLE β ON THE BF CHARACTERISTICS

Computed results of the dependence of the free spectral range of the BF, the tuning factor $d\lambda/d\varphi$, and the factor $d\lambda/d\beta$ on the angle β are given in Table II. The characteristics are given for a filter with the thickness of the first plate, for instance, of $h_1 = 1.86$ mm and arbitrary integral ratio of plate thickness for $\varphi = 40°$. The angles β given in Table II are characterized by the fact that for $\beta = \beta^*$, the wavelength of the BF transmission peak is 595 nm without changing h_1 and φ. The free spectral ranges $\Delta\lambda_k$ and $\Delta\lambda_g$ are calculated based on the fundamental transmission peak at a chosen wavelength and are equal to the spectral distances to the next fundamental peaks of the $T(\lambda)$ function in the short-wavelength and long-wavelength regions of the spectrum, respectively. To determine the BF tuning factor, a shift in the transmission peak is found at $\lambda = 595$ nm by varying the angle of tilt φ by 1°. The relation between the factors $d\lambda/d\beta$ and $d\lambda/d\varphi$ derived from the data of Ref. 6 is used to determine arbitrary $d\lambda/d\beta$,

$$\frac{d\lambda}{d\beta} = \frac{d\lambda}{d\varphi}\left(\frac{\tan\beta}{\tan\varphi} - \frac{1}{\sin\varphi \tan\theta}\right).$$

This computation technique allows us to make a correct comparison between BF characteristics with different angles β.

Table II shows that the maximum of the BF tuning factor is reached at $\beta = 0°$. The angle γ between the optic axis and the normal of the extraordinary wave decreases with increasing angle β. This leads to an increase in the derivative $d\gamma/d\varphi$, and this explains the increase in the tuning factor. We note that the factors $d\lambda/d\varphi$ and $d\lambda/d\beta$ are independent of plate thicknesses and are determined by the angle β and, to a negligible extent, by the orientation of the BF.[6] An increase in the angle β for a fixed thickness of the first plate may lead to significant increase (4–7 times) of the BF free spectral range.

USE OF CALCULATED DATA IN BF DESIGN

In designing BF for a tunable laser, it is necessary to proceed from the allowable reduction in the contrast function $T(\lambda)$ at the edge of the tuning region. Assuming, for instance, the maximum deterioration of the contrast func-

TABLE II.

β^* (deg)	$\Delta\lambda_k$ (nm)	$\Delta\lambda_g$ (nm)	$\frac{d\lambda}{d\varphi}$ (nm/deg)	$\frac{d\lambda}{d\beta}$ (nm/deg)
0	19.0	20.2	3.3	−7.3
8.7	22.0	23.8	5.0	−11.2
16.6	26.2	28.6	6.9	−14.2
24.3	32.0	36.6	9.5	−17.9
32.5	41.4	49.8	12.9	−21.4
42.3	60.6	78.6	18.4	−24.5
53.5	92.0	139.4	25.5	−20.6

tion $T(\lambda)$ to be a factor of 2, we find from Fig. 3 that variation in the angle of tilt $\Delta\varphi = |\varphi - \varphi_{opt}|$ should not exceed 14°. Consequently, by varying the angle φ by $\Delta\varphi = 14°$, the filter should tune the wavelength of the transmission peak by no less than half the required tuning band. By denoting the spectral width of the required tuning band as $\Delta\lambda^p$, we have the condition for the factor $d\lambda/d\varphi$

$$\frac{d\lambda}{d\varphi} > \frac{1}{2}\frac{\Delta\lambda^p}{\Delta\varphi}. \qquad (8)$$

We find the allowed angles β from Table II using the tuning factor. We obtain φ_{opt} from Eq. (7) by choosing the angle β. For given wavelength of the center of the transmission band λ^*, angles β and φ_{opt}, we find the index of refraction of the extraordinary wave n from Eq. (3) or (3a) and the thickness of the first plate of the BF from the formula[10]

$$h_1(m) = \frac{m\lambda^*}{(n^2(\beta, \varphi_{opt}, \lambda^*) - \sin^2 i)^{1/2} - (n_0^2(\lambda^*) - \sin^2 i)^{1/2}}, \qquad (9)$$

where m is an integer. With the desired thickness $h_1(m)$, the free spectral range $\Delta\lambda_k$ should be wider than the required tuning band.

Thus, in BF design, there is certain degree of freedom in the selection of the angle β and thickness $h_1(m)$. The angle β is limited only from below and, for example, for $d\lambda/d\varphi > 5$ nm/deg, the BF can have any angle β that satisfies the inequality $\beta \gtrsim 9°$ (Table II).[1] Calculation by Eq. (9) can also give precisely several thicknesses $h_1(m_1),...,h_1(m_N)$, for which $\Delta\lambda_k$ (and this also means $\Delta\lambda_g$) will be wider than the tuning band. Let us consider which angles β and thicknesses $h_1(m_N)$ we should select.

We recommend the selection of β and $h_1(m_N)$ for the purpose of efficient use of adjacent and succeeding fundamental peaks of the transmission function $T(\lambda)$ when operating the filter in those regions of the spectrum, where these peaks are situated at $\varphi = \varphi_{opt}$. The BF transmission function is periodic and possesses only smaller $\Delta\lambda_k$ and $\Delta\lambda_g$ in the shorter-wavelength region of the spectrum. The design formulas for $T(\lambda)$ allows us to find almost exactly the spectral position of all the fundamental transmission peaks of any BF. By choosing β and $h_1(m_N)$ in such a manner that the wavelengths of certain fundamental transmission peaks would be close to the wavelengths of the peaks of the gain curves of certain dyes, we can broaden significantly the functional capabilities of the BF. In this case, lasing in the majority of dye lasers will take place sucessively at different fundamental transmission peaks. For the majority of the dyes, the BF will be tuned by varying the angle φ near φ_{opt}. Thus, for example, a BF with $h_1 = 398$ μm, $\beta = 0°$, $\varphi_{opt} = 40°$ has main transparency peaks at 415 nm [for $m = 9$, see Eq. (9)]. 459 nm ($m = 8$), 517 nm ($m = 7$), 595 nm ($m = 6$), 705 nm ($m = 5$). We find from Ref. 13 that this BF can tune the wavelength of six dye lasers; stilbene 1 (415), cumarin 2 (454), cumarin 47 (469), cumarin 30 (510), rhodamine 6G (593), and puridin 1 (710). The wavelength of the peak in the spectral distribution of the output power of the laser of a given dye pumped by a high power ion laser is given in nanometers in parentheses. The overal tuning band of the BF in the six dyes is ~350 nm.[13] On the other hand, in order to operate with these six dyes, two or three BF are required (for blue, green-yellow, and red emissions[14]), designed for using one fundamental transmission peak and having a limited tuning band (of the order of 100 nm for the BF, commercially manufactured by Coherent, Inc.[15]). Thus, the advantage of the BF so designed as to include the potential of a tunable laser with different active media lasing successively at different fundamental peaks of the $T(\lambda)$ function are clear.

We note that the design formulas for the $T(\lambda)$ function can be applied to numerical modeling of both tunable lasers using BF, and cw dye and color center lasers, and pulsed dye lasers with a large effective number of passes.

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[1] We should, however, take into account that overestimation of the angle β leads to deterioration of the accuracy and reproducibility of BF tuning.

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